Econ 802

First Midterm Exam

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All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. Consider a technology with one input ($y_1 \leq 0$) and one output ($y_2 \geq 0$). The firm can choose from the following production plans. If $-2 \leq y_1 \leq 0$ then $y_2 = 0$. If $y_1 < -2$ then y_2 must satisfy $0 \leq y_2 \leq -y_1 - 2$.

(a) Draw a graph of the production possibilities set Y. Clearly label the axes and the set itself. Is Y convex? Is Y strictly convex? Explain.

(b) Consider the problem of maximizing profit py subject to $y \in Y$. For what price vectors $p = (p_1,p_2) > 0$, if any, does this problem have a solution? Explain using a graph of Y and isoprofit lines of the form $\pi = p_1y_1 + p_2y_2$ where π is a constant.

(c) Does the production function derived from Y have decreasing returns, constant returns, increasing returns, or none of these? Justify your answer using the definition of each concept.

2. Let $\pi(p,w)$ be the profit function for a firm, where p is a scalar indicating output price and w is a vector of n input prices.

(a) Prove that $\pi(p,w)$ is homogeneous of degree 1.

(b) Give an economic interpretation of the expression $p\pi_p(p,w) + \sum_{i=1..n} w_i\pi_i(p,w)$, where π_p is the derivative with respect to p and π_i is the derivative with respect to w_i. If possible, relate your answer to the result in part (a).

(c) Fix w (it is held constant during this question). Then consider the identity $\pi_L(p) \equiv \pi_S[p,z(p)]$ where $\pi_L(p)$ is long run profit at the price p, $\pi_S(p,z)$ is short run profit at price p when the firm has z units of a fixed input on hand, and z(p) is the amount of the fixed input the firm would choose in the long run at price p. What can you say about the sign of the partial derivative of π_S with respect to z, evaluated at the point [p,z(p)]? Explain.

3. The firm's production function is the scalar product f(x) = ax where $a = (a_1 .. a_n) > 0$ is a vector of positive constants and $x = (x_1 .. x_n) \geq 0$ is the input vector.

(a) Is this function concave? Strictly concave? Does its Hessian matrix satisfy the usual necessary second order condition for profit maximization? Will it satisfy the usual sufficient SOC? Justify your answer in each case.

(b) Does the firm's cost minimization problem always have a solution? Explain. Hint: the Weierstrasse Theorem says that a continuous function defined on a closed and bounded set must achieve a minimum at some point in the set.

(c) What must be true about the ratio w_i/a_i if the firm uses a positive amount of input i? Justify your answer using Kuhn-Tucker multipliers.

4. In period t = 1 we observe the input price vector $w^1 = (1,3)$ and the input vector $x^1 = (5,2)$. In period t = 2 we observe the input price vector $w^2 = (8,4)$ and the input vector $x^2 = (2,4)$.

(a) Could these observations have come from a cost-minimizing firm? Explain.

(b) Suppose input requirement sets V(y) must be convex and monotonic. Draw a graph showing the smallest possible set VI that is consistent with the data, and then draw a second graph showing the largest possible set VO that is consistent with the data. Carefully label all points and indicate the slopes of all lines or line segments, and explain your reasoning.

(c) Suppose in period t = 3 we observe the input vector (1,6). Does this rule out VI as a possible V(y) set? Does it rule out VO? Given this additional observation, what (if anything) can you say about the input prices in period 3?

5. A firm has the production function $f(x) = x_1^\alpha + x_2^\beta$ where $\alpha > 0$ and $\beta > 0$.

(a) Compute the elasticity with respect to scale e(x). What must be true about α and β if $e(x) > 1$ for all x? What must be true if $e(x) < 1$ for all x? Explain.

(b) Now assume $\alpha = \beta$. Write down the first order conditions that must hold if $x^* > 0$ solves the cost minimization problem. You can ignore corner solutions, and you don't need to solve explicitly for x*; just state the FOC. Then state a <u>sufficient</u> second order condition for x* to be a solution. Explain using a graph.

(c) Derive the cost function c(w,y) for the case where the sufficient SOC in part (b) does <u>not</u> hold. Is this cost function differentiable with respect to w? Explain.